**Mathematical Foundations**

**Instructions**

Please share your answers wherever applicable in-line with the word document. Submit code separately wherever applicable. Mathematical calculations which are manually performed should be uploaded with a picture along with the explanation in a word document.

Please ensure you update all the details:

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**Topic: Mathematical Foundations**

**Note: Submit pictures of mathematical calculations**

**Problem Statements**

**Q1) Find the maximum and minimum values of the function: x3 - 3x2 - 9x + 12?**

To find the maximum and minimum values of the function

f(x)=x³−3x²−9x+12, we can follow these steps:

**Step 1:** Find the first derivative 𝑓′(𝑥)

The first derivative helps us find the critical points, where the function could have maxima or minima.

f′=d/dx(x³−3x²−9x+12)=3x²−6x−9

**Step 2:** Set the first derivative equal to zero to find critical points

3x²−6x−9=0

Divide by 3 to simplify:

x²−2x−3=0

Now, factor the quadratic equation:

(x−3)(x+1)=0

So, the critical points are: x=3 and x=−1

**Step 3:** Find the second derivative f ′′(x)

The second derivative will help determine the nature of the critical points (maxima or minima).

f′′(x)= d/dx(3x²−6x−9)=6x−6

**Step 4:** Determine the nature of the critical points

At x=3:

f′′(3)=6(3)−6=18−6=12>0

Since

𝑓′′(3)>0,x=3 is a local minimum.

At x=-1

f′′(-1)=6(-1)−6=-6−6=12<0

Since

f′′(−1)<0, x=−1 is a local maximum.

**Step 5:** Calculate the function values at the critical points

Now, substitute the critical points back into the original function f(x):

At x=3:

f(3) = 3³-3(3²)-9(3)+12=27-27-27+12 = -15

At x=-1:

f(-1) = (-1)³-3(-1)²-9(-1)+12 =-1-3+9+12=17

Conclusion

The local maximum value is f(−1)=17.

The local minimum value is f(3)=−15.

**Q2) Calculate the slope and the equation of a line which passes through the points (-1, -1), (3, 8)**

To find the slope and equation of the line passing through the points (−1,−1) and (3,8), follow these steps:

**Step 1: Find the slope m**

The formula for the slope of a line passing through two points (x1,y1) and (x2,y2) is:

Substitute the given points (−1,−1) and (3,8):

m= = =

slope is :- m =

**Step 2: Use the point-slope form to find the equation of the line**

The point-slope form of the equation of a line is:

y−y1=m(x−x1)

Substitute m= m and one of the points, say (−1,−1):

y−(−1)= (x−(−1))

Simplify:

y+1=(x+1)

**Step 3: Simplify the equation**

Expand the equation:

y+1= x+

Now, subtract 1 from both sides:

y= x+ −1

Convert 1 to a fraction with denominator 4:

y= x+ −

Simplify:

y= x+

**Final Equation**

The equation of the line is:

y= x+

**Q3) Solve for w’(z) when**



To solve for w′(z), where w(z) = ,we'll use the quotient rule for differentiation.

**Quotient Rule:**

If you have a function w(z)= , then the derivative is given by:

w′(z) =

**Step 1: Identify f(z) and g(z)**

For the given function:

F(z) = 4z-5, g(z) = 2 – z

Step 2: Compute f′(z) and g′(z)

* f′(z)=4 (since the derivative of 4z−5 is 4)
* g′(z)=−1 (since the derivative of 2−z is -1)

**Step 3: Apply the quotient rule**

Substitute the values into the quotient rule formula:

w′(z) =

**Step 4: Simplify the expression**

Expand and simplify:

w′(z) =

w′(z) =

**Final Answer**

So, the derivative w′(z)w'(z)w′(z) is:

w′(z) =

**Q4) Consider Y = 2x3+6x2+3x. Identify the critical values and verify if it is the maxima or minima.**

To find the critical values and determine whether they are maxima or minima for the function:

Y=2x³+6x²+3x

**Step 1: Find the first derivative Y′(x)**

To locate critical values, we first compute the first derivative of Y:

Y′(x)=(2x³+6x²+3x) = 6x²+12x+3

**Step 2: Set the first derivative equal to zero**

To find the critical points, set the derivative equal to zero and solve for x:

6x²+12x+3=0

**Step 3: Solve the quadratic equation**

We'll solve the quadratic equation using the quadratic formula:

For the equation 6x²+12x+3=0, a=6, b=12, and c=3. Substitute these values into the quadratic formula:

So, the critical values are:

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**Step 4: Verify maxima or minima using the second derivative**

To verify if these critical points correspond to maxima or minima, we compute the second derivative of Y;

Now evaluate Y′′(x)Y''(x)Y′′(x) at the critical points:

For

This expression needs to be evaluated numerically to check the sign.

For

* *Similarly, evaluate numerically to check if it's positive or negative.*

*Once you evaluate the second derivative at these points, the sign will determine whether the critical values correspond to maxima (if negative) or minima (if positive).*

**Q5) Determine the critical points and obtain relative minima or maxima of a function defined by**



The given function is:

**Step 1: Find the partial derivatives**

To find the critical points, we need to calculate the partial derivatives of y with respect to x1 and x2 and set them equal to zero.

Partial derivative with respect to x1:

Partial derivative with respect to x2​:

**Step 2: Set the partial derivatives equal to zero**

We now set both partial derivatives equal to zero to find the critical points.

### Step 3: Solve the system of equations

#### From equation (2):

= 0 => -

Substitute - into equation (1):

Now , substitute into

x1​=−2(1)=−2

Thus, the critical point is (x1,x2)=(−2,1).

**Step 4: Find the second partial derivatives to determine maxima or minima**

To determine whether this critical point is a relative maximum, minimum, or a saddle point, we compute the second partial derivatives.

**Second partial derivatives:**

**Step 5: Use the second derivative test**

The second derivative test involves computing the discriminant D, which is given by:

D =

Substitute the values of the second partial derivatives:

D = (4)(4) –(2)² = 16 – 4 = 12

Since D>0 and > 0,the function has a **relative minimum** at the critical point (−2,1)(-2, 1)(−2,1).

**Conclusion**

The function has a relative minimum at (x1,x2)=(−2,1)